positioning the sources in a sufficiently large distance from discontinuities is reduced considerably.

In principle, the method presented includes the possibility for calculating planar stripline structures, where the permittivity of the substrate is given by $\epsilon = \epsilon(x)$. In that case, the partial differential equations for the scalar potentials are of the Sturm–Liouville type [6].

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Short Papers

High-Order Mode Cutoff In Rectangular Striplines

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***Abstract*—The higher order mode cutoff characteristics of rectangular stripline structures, with thin center conductors, are discussed. Data are given, using an alternative method of presentation, on the normalized cutoff of the first eleven higher order modes. Discussions are included on the physical reasons why cutoff is altered for some modes, relative to that in rectangular waveguides, but not for others.**

I. Introduction

Large-scale rectangular strip-transmission lines containing a propagating transverse electromagnetic (TEM) field are now widely used for such purposes as electromagnetic susceptibility and emissions testing, calibration of field probes and survey meters, and studies on the biological effects of radiofrequency (RF) radiation exposure. These structures are characterized by an air dielectric and a thin center conductor (septum) surrounded by a rectangularly shaped shield. This provides for an optimally sized test space within the line in which equipment, field probes, or experimental animals, etc., are exposed to a well-defined and reasonably uniform field. Crawford [1] has discussed the properties of such lines and has described a family of TEM "cells" constructed at the National Bureau of Standards. These devices are commercially available and have been termed "Crawford Cells" or "TEM Transmission Cells" by their manufacturers.

The usable frequency range of these devices is of obvious importance to those involved in their use. Whereas it had been thought that these structures could not be used above the cutoff frequency where the first higher order mode is predicted to occur [2], it has recently been shown by Hill [3] that such is not necessarily the case. In his important study, Hill has shown that significant perturbation of the internal fields within the structure exists primarily at certain discrete frequencies where resonances of the higher order mode fields occur. Such resonances will occur when the equivalent electrical length of the strip-transmission line

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$l_{(mn)}$ is equivalent to multiples of a half-guide-wavelength $\lambda_{g(mn)}/2$, for the particular higher order mode being considered, i.e.

$$l_{(mn)} = p\lambda_{g(mn)}/2; \; p = 1,2,3,\cdots. \tag{1}$$

The subscripts m, n denote the higher order mode. Substituting (1) into the well-known relationship for wavelength in waveguides

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_{c^2(mn)}} \tag{2}$$

where $\lambda_{c(mn)}$ represents the cutoff wavelength value, gives an expression by which the various resonant frequencies $f_{R(mnp)}$ can be predicted

$$f^2_{R(mnp)} = f^2_{c(mn)} + \left(\frac{pc}{2l_{(mn)}}\right)^2 \tag{3}$$

where $f_{c(mn)} = \frac{c}{\lambda_{c(mn)}}$; '$c$' is the velocity of light.

Note that the equivalent electrical length $l_{(mn)}$ given in (3) generally exceeds the actual physical length of the transmission line due to the presence of fringing fields at the line terminations. The magnitude of this difference varies with the mode being excited as well as the cross-sectional dimensions of the line and depends on whether the termination is abrupt (i.e., the line has a box-like shape with square ends) or gradual (tapered ends). Hill [3] was able to derive empirical values of l_{mn} for two different tapered cells, based on measured values of the resonant frequency $f_{R(mnp)}$. Attempts at predicting the fringing field correction are presently being undertaken in order to confirm the accuracy of Hill's empirical estimates.

To what extent these structures are usable when higher order mode resonances are present and whether or not they are usable at frequencies between such resonances depends very much on the particular application for which the transmission line is being used and the manner in which it is loaded (i.e., the composition and size of the object placed in the line). Some modes have been shown to interact strongly with any sizable load within the line while others interact little, owing to the differing field patterns of these modes. For some applications such as, for example, field-probe calibrations, it is possible to correct for the presence of the higher order mode fields by alternately positioning the probe on both sides of the center plate and averaging the two response curves versus frequency.

Accurate prediction of the various resonant frequencies in rectangular stripline structures using (3) requires a knowledge of the cutoff frequency $f_{c(mn)}$ for a number of the first higher order modes that can propagate in such structures. The purpose of this short paper is to review some of the existing data on this subject, as well as to present some additional data, as yet unpublished, in a form that is readily usable by those working with TEM-mode cells.

II. Cutoff Data

The higher order mode problem in rectangular coaxial structures (i.e., those with a center conductor of appreciable thickness) has been independently studied by Brackelmann *et al.* [4] and Gruner [5]. Baier [6] published additional data on cutoff in rectangular coaxial lines of varying dimensional parameters. More recently, Gruner [7] published data on the TE_{01} mode cutoff in rectangular lines with thin center conductors ($t/b \leq 0.1$ where 't' is the conductor thickness and 'b' is the vertical side-wall dimension; see Fig. 1). Details of the numerical techniques employed are provided in both of Gruner's papers and will not be further

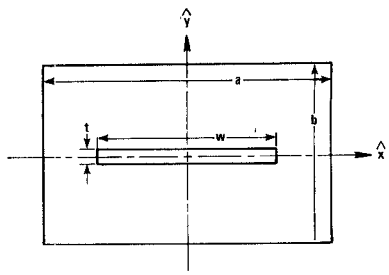

Fig. 1. Cross section of rectangular stripline structure.

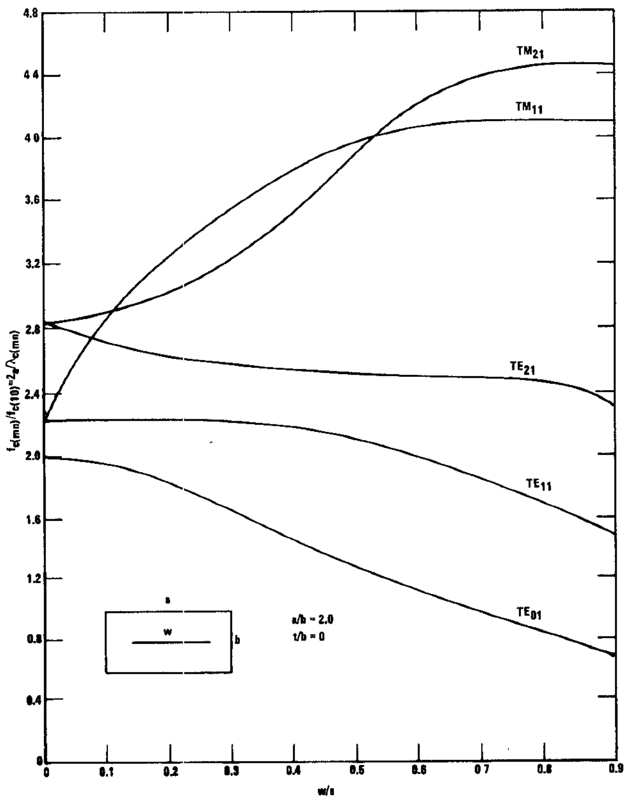

Fig. 2 Normalized cutoff frequency versus the parameter w/a for five of the altered modes (case $a/b = 2.0$, $t/b = 0$).

elaborated on here. Similar data for the TE_{11}, TM_{11}, and TM_{21} modes were published by Tippet and Chang [9] in a NBS report that has not been widely disseminated.

It has been shown [7], [9] that, for rectangular structures having a zero-thickness center conductor, the cutoff frequency for all modes with n-odd subscripts will be altered relative to that of its rectangular waveguide counterpart where no center conductor is present; i.e.

$$TE_{m,2n+1} \quad (m,n = 0,1,2\cdots)$$
$$TM_{m,2n-1} \quad (m,n = 1,2,3\cdots).$$

Cutoff for all of the remaining modes having n-even subscripts

$$TE_{m,2n} \quad (m,n = 0,1,2\cdots, \; m \neq n \neq 0)$$
$$TM_{m,2n} \quad (m,n = 1,2,3\cdots)$$

remain unchanged relative to that of their waveguide counterpart. The physical reasons for this effect are discussed in the next section. For the unaltered modes, the normalized cutoff frequency, relative to that of the dominant TE_{10} mode, can be conveniently

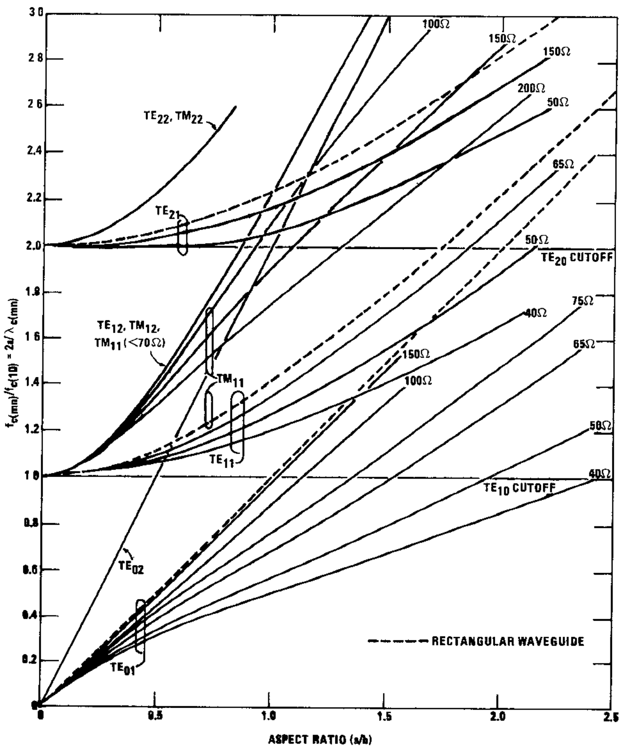

Fig. 3. Normalized cutoff frequency versus a/b for the lowest higher order modes.

expressed as follows:

$$\frac{f_{c(mn)}}{f_{c(10)}} = \frac{2a}{\lambda_{c(mn)}} = \sqrt{m^2 + n^2(a/b)^2} \tag{4}$$

where 'a' is the horizontal width dimension; see Fig. 1. Of the modes where cutoff values are altered, the first four are of particular significance, namely the TE_{01}, TE_{11}, TE_{21}, and TM_{11} modes. Fig. 2 shows the normalized frequency cutoff $(f_{c(mn)}/f_{c(10)})$ versus the dimensional ratio w/a, where 'w' is the horizontal width of the center plate for five of the altered modes. These curves are shown for an aspect ratio value of $a/\text{ib} = 2.0$; changing the aspect ratio significantly alters the normalized cutoff values.

The curves of Fig. 2 represent the traditional manner in which cutoff data have been presented. This method of presentation is not optimal because, in practice, most rectangular striplines possess a fixed characteristic impedance, Z_0 (usually 50 Ω) but use differing aspect ratio values that cover the range $0.5 < a/b < 2$. It is felt that the data can be better interpreted if the normalized cutoff data are presented as a function of the aspect ratio over the range $0 \leqslant a/b \leqslant 2$ for various chosen values of Z_0. This has been done in Fig. 3 which shows normalized cutoff data for the four altered modes; seven of the unaltered modes are also included for comparison purposes. The dotted lines represent the limiting case for a rectangular waveguide with no center strip present. Additional higher order modes such as, for example, the TE_{03}, $TE_{04}, \cdots, TE_{13}, TE_{14}, \cdots, TE_{23}, TE_{24}, \cdots$, etc., are only significant for structures with $a/b < 0.5$ and have been omitted from Fig. 3 in order to avoid excessive detail. The curves of Fig. 3 were generated using the characteristic impedance data shown in Fig. 4, i.e., the required value of w/a could be determined for given values of Z_0 and a/b. The data of Fig. 4 were obtained

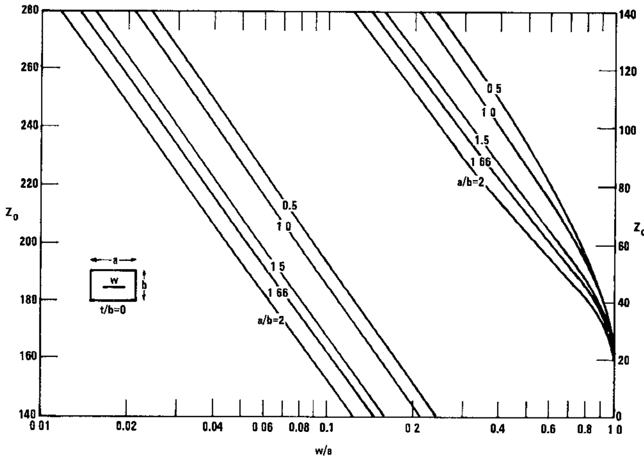

Fig. 4. Characteristic impedance, Z_0 versus w/a for various a/b values (case $t/b = 0$).

using the technique described by Weil and includes a correction for the edge-interaction effect [10]. Note that the data given in Figs. 3 and 4 of Tippet and Chang [9] for the TE_{11} and TM_{11} cutoff agree well with the data shown in Fig. 3.

III. Discussion

The data presented in Fig. 3 show well how the presence of the center conductor in the rectangular structure alters the cutoff of certain modes relative to that in rectangular waveguide. It can be clearly seen that, for the three altered TE-modes (TE_{01}, TE_{11}, and TE_{21}), the cutoff frequencies are reduced, with the reduction increasing markedly as the characteristic impedance value is lowered. It is evident that, for a 50-Ω line, the first higher order mode is always the TE_{01} for $a/b \leqslant 1.94$.

Such effects are physically explainable as follows: for all of the TE-modes having even numbered values of the y-axis subscript 'n', there exists an electrical wall along the x-axis, parallel to the center conductor (see Fig. 1). This means that for the n-even case, the E-field level is zero along the x-axis and no capacitive coupling can exist between the center-strip edges and the vertical side walls. Consequently, the presence of the center strip does not alter cutoff for the n-even modes. For the TE-modes having n-odd subscripts, however, the reverse situation exists. A magnetic wall, where maximum E-field exists, is present along the x-axis. Capacitive coupling now exists between the center-strip edges and the vertical side walls that causes the structure to appear electrically larger in the vertical b-dimension than its actual physical dimension. Consequently, cutoff frequencies are lowered for these modes relative to their rectangular waveguide counterparts, as seen in Fig. 3.

The electrical enlargement of rectangular structures in the b-dimension is a useful concept for explaining the changes in the cutoff characteristics of the altered TE-modes. If b' represents the enlarged dimension (i.e., the vertical height of the equivalent rectangular waveguide without center strip), then, for the TE_{01} mode, the normalized cutoff frequency is given by

$$\frac{2a}{\lambda_{c(01)}} = \frac{a}{b'}. \tag{5}$$

By inverting the TE_{01} cutoff data in Fig. 3, a plot of the normalized cutoff wavelength against b/a is obtainable, i.e., $\lambda_{c(01)}/2a = b'/a$ against b/a.

For lines with very low characteristic impedance ($Z_0 \rightarrow 0$), the center plate occupies an increasingly larger proportion of the rectangular width, a of the structure. For this case, the center strip coupling becomes large so that $b' \gg b$ and $a/b' \rightarrow 0$. Hence, from (4), it is evident that, for this case, $2a/\lambda_{c(01)} \rightarrow 0$ for all a/b. Similarly, for the cases of the TE_{11} and TE_{21} modes, it can be seen that $2a/\lambda_{c(11)} \rightarrow 1$ and $2a/\lambda_{c(21)} \rightarrow 2$ for all a/b. Both Gruner's and Baier's results for the rectangular coaxial line [5], [6] confirm the above.

Referring back to Fig. 3 again, it is apparent that, for the only altered TM-mode shown (TM_{11}), the cutoff frequency is increased. Whereas when no center conductor is present (waveguide case), the TM_{11} mode will always propagate before the TE_{21} mode, this situation generally becomes reversed when the center strip is present. For the case of a 50-Ω line, it is apparent that the TE_{21} cutoff is below that of the TM_{11} mode for all $a/b \geqslant 0.9$. Note that the presence of a relatively narrow center strip ($w/a < 0.2$) causes a marked increase in the TM_{11} cutoff, but that this increase does not exceed that corresponding to the TM_{12} cutoff. In fact, for lines with $Z_0 < \sim 70$ Ω, the TM_{11} cutoff is essentially the same as that for the TM_{12} mode. In this case, when the center conductor occupies an appreciable fraction of the width ($0.6a$ or more), it apparently acts as an electrical wall, causing the TM_{11} mode field structure to break up into a TM_{12} structure that contains an H-field null along the x-axis. These results are confirmed by Gruner's data [5] which show the curves for the TM_{11} and TM_{12} cutoff, as well as those for the TM_{21} and TM_{22} modes merging for values of $w/a > 0.6$.

Acknowledgment

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Field Patterns and Resonant Frequencies of High-Order Modes in an Open Resonator

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Abstract —**Using the electromagnetic perturbation theory, it is shown that the linearly polarized TEM_{pl} modes ($l > 0$) predicted by conventional methods are not the resonant modes in an open resonator. Instead, two other series of high-order modes are proposed with improved accuracy in resonant frequencies.**

I. Nomenclature

c	velocity of light,
D	distance of separation between reflectors,
$\bar{E}$	electric field strength,
E_x	$\psi(\rho, \theta, z) \cdot \exp(-jkz)$,
f	resonant frequency,
j	$\sqrt{-1}$,
k	propagation constant in free space,
l	azimuthal mode number,
$L_p^l(x)$	generalized Laguerre polynomial,
$L_p^{l\prime}(x)$	$(d/dx) L_p^l(x)$,
p	radial mode number,
q	axial mode number,
R	radius of curvature of the phase front,
R_1	radius of curvature of the reflector,
w	radius of the beam wave,
w_0	radius of the beam waist,
w_1	radius of the beam wave at $z = D/2$,
W	energy stored,
$\hat{z}$	unit vectors along the z direction,
Δ	small increment,
ρ, θ, z	cylindrical coordinates,
$\Phi = \arctan(z/z_0)$	additional phase shift.

II. Introduction

From the approximate beam-wave theory [1], there exists a complete set of linearly polarized Gaussian beam modes, which are conventionally designated as TEM_{pl}. These modes can be separated into two series, and can be represented by

$$E_x = \left(\sqrt{2}\,\frac{\rho}{w}\right)^l L_p^l\left(\frac{2\rho^2}{w^2}\right)\frac{w_0}{w}\exp\left(\frac{-\rho^2}{w^2}\right) \cdot \exp\left[-jkz + j(2p+l+1)\Phi - j\frac{k\rho^2}{2R}\right]\cos l\theta \quad (1)$$

and

$$E_x = \left(\sqrt{2}\,\frac{\rho}{w}\right)^l L_p^l\left(\frac{2\rho^2}{w^2}\right)\frac{w_0}{w}\exp\left(\frac{-\rho^2}{w^2}\right) \cdot \exp\left[-jkz + j(2p+l+1)\Phi - j\frac{k\rho^2}{2R}\right]\sin l\theta \quad (2)$$

where p and l are the radial and azimuthal mode numbers, respectively. By combining two linearly polarized modes of the same order, it is possible to synthesize other polarized modes in

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